Geometry, Quarter 2, Unit 2.1

Proving Triangles are Similar

Overview

Number of instructional days: 15 (1 day = 45–60 minutes)

Content to be learned

- Experiment with properties of dilations, given the center and scale factor.
- Experiment with the dilation of a line segment.
- Prove theorems of line segments to be longer or shorter in the ratio given by the scale factor.
- Experiment with properties of similarity transformation.
- Prove theorems about triangles.
- Prove relationships in geometric figures.
- Prove congruence and similarity criteria for triangles to solve problems.

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Use transformations in algebraic expressions.
- Use conjectures about the form and meaning of a solution and plan a pathway for that solution.
- Create a table to prove an algebraic expression.

Construct viable arguments and critique the reasoning of others.

- Write a paragraph proof that signifies triangle similarities.
- Graph triangles on a Cartesian plane to determine triangle similarity.

Model with mathematics.

- Use a graphing calculator to determine similarity.
- Use constructions with triangles to determine similarity.
- Use chart paper to illustrate triangle similarity to solve problems.

Look for and express regularity in repeated reasoning.

- Use definitions, postulates, and theorems to evaluate triangle similarities.

Essential questions

- What did you learn about triangle similarities after experimenting with them in a coordinate plane?
- What are the similarities and differences of special triangles?
- How do you determine whether two triangles are similar?
- How do you use coordinate geometry to find relationships within triangles?
- When can you use special triangles to solve problems in the real world?
- How does rigid motion relate to triangle similarity?

Written Curriculum

Common Core State Standards for Mathematical Content

Similarity, Right Triangles, and Trigonometry	G-SRT

Understand similarity in terms of similarity transformations.

G-SRT.1 Verify experimentally the properties of dilations given by a center and a scale factor:

a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.

b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

G-SRT.2 Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

G-SRT.3 Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

Prove theorems involving similarity.

G-SRT.4 Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.

G-SRT.5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

Common Core State Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Clarifying the Standards

Prior Learning

In grade 4, students identified and drew lines and angles. They were introduced to right, acute, and obtuse triangles. In seventh grade, students focused on constructing triangles from three measures of angles or sides, noticing conditions that determine a unique triangle. Eighth-grade students understood when a two-dimensional figure was similar to a second figure.

Current Learning

Students understand terms of similarity transformations, prove theorems involving similarity, define trigonometric ratios, and solve problems involving right triangles.

Future Learning

In algebra 2, students will prove and apply trigonometric identities. They will use special triangles to determine geometrically the value of sine, cosine, and tangent. Students will be able to use the knowledge of similar triangles in the fields of construction management, engineering, natural sciences, advertising, marketing, promotion, public relations, and sales.

Additional Findings

In this unit, building on prior knowledge of the Pythagorean Theorem, students use a rectangular coordinate system to verify geometric relationships. This will include properties of special right triangles. Students will continue their study of quadratics by predicting the geometric and algebraic definitions of a parabola.

Geometry, Quarter 2, Unit 2.2

Solving Right Triangles Using Trigonometric Ratios

Overview

Number of instructional days: 14 (1 day = 45–60 minutes)

Content to be learned

- Define trigonometric ratios.
- Prove side ratios in right triangles as properties of the angles in a triangle.
- Perform geometric constructions.
- Explore the relationship between the sine and cosine and complementary angles.
- Use the Pythagorean Theorem to solve right triangles and in application problems.
- Use trigonometric ratios to make formal geometric constructions.

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Discuss a trigonometric ratio and predict the outcome.
- Graph special triangles to find side lengths (Pythagorean Theorem).

Model with mathematics.

- Prove trigonometric ratios using constructions.
- Use a graphing calculator to solve right triangles in applied problems.

Essential questions

- How do geometric models describe sine and cosine relationships?
- How can a coordinate grid be used to model and describe trigonometric ratios?
- How can proofs be used to solve geometric problems?
- What is trigonometry and how are right triangles used in it?
- How do you show that two triangles are similar?
- How do you use coordinate geometry to find relationships within triangles?

Written Curriculum

Common Core State Standards for Mathematical Content

Similarity, Right Triangles, and Trigonometry	G-SRT

Define trigonometric ratios and solve problems involving right triangles.

G-SRT.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

G-SRT.7 Explain and use the relationship between the sine and cosine of complementary angles.

G-SRT.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.★

Common Core State Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Clarifying the Standards

Prior Learning

In first grade, students composed two-dimensional shapes such as triangles, rectangles, and squares. In the fourth grade, students recognized right triangles as a category and identified right triangles. Sixth-grade students computed the area of right triangles and other triangles. In eighth grade, students explained and used a proof of the Pythagorean Theorem and its converse.

Current Learning

In this unit, students use a rectangular coordinate system to verify geometric relationships, including properties of special triangles and quadrilaterals.

Future Learning

In Algebra II, students will expand the Pythagorean Theorem using trigonometric identities. They will use the coordinate plane to extend trigonometry to model periodic phenomena. Students will continue to use theses knowledge and skills in a number of real-world professions, including funeral directors, farmers, ranchers, agricultural managers, architects (excluding landscaping and naval), and engineering and natural science managers.

Additional Findings

The work in this unit is for students to begin to formalize their geometric experiences. They will use more precise definitions and develop careful proofs. Geometric shapes can be described by equations, making algebraic manipulations a tool for geometry understanding, modeling, and proofs.

Geometry, Quarter 2, Unit 2.3

Using Descriptions of Rigid Motion to Transform Geometric Figures

Overview

Number of instructional days: 11 (1 day = 45–60 minutes)

Content to be learned

- Explore transformations of points in a plane.
- Explore transformations of rectangles in a plane.
- Explore transformations of parallelograms in a plane.
- Explore transformations of trapezoids in a plane.
- Explore transformations of other polygons in a plane.
- Prove definitions of rotations, reflections, and transformations in terms of angles.
- Transform geometric figures using rotation, reflection, and transformations.
- Prove theorems about parallelograms.
- Prove theorems of rigid motion in terms of congruency.

Mathematical practices to be integrated

Reason abstractly and quantitatively.

- Discuss the theorems needed to determine parallelogram congruency.
- Use geometric descriptions of rigid motion to predict the effect of a given figure.

Model with mathematics.

- Use constructions to draw transformed figures.
- Construct rectangles, parallelograms, and trapezoids to describe rotations and reflections.

Use appropriate tools strategically.

- Use dynamic geometry software to show rotation, reflection, and translation.
- Use transparencies to describe transformations as a function.

Attend to precision.

- Illustrate the methods for using graph paper to determine rotation, reflection, and translation.
- Determine the congruency of two triangles in terms of rotation on a coordinate plane.

Essential questions

- What is the difference between the rotation and the reflection of a figure?
- What is the definition of congruency of two triangles in terms of rigid motion?
- What methods are used to transform geometric figures to describe rotations?
- How does rigid motion relate to triangle congruency?
- How can transformations be used to describe geometric rotation, reflection, and translation?
- How can describing and classifying rectangles, parallelograms, trapezoids, and other regular polygons be useful in solving geometric problems in our 3-D world?
- In a parallelogram, how would you prove that opposite sides are congruent, opposite angles are congruent, and the diagonals bisect each other?

Written Curriculum

Common Core State Standards for Mathematical Content

Congruence	**G-CO**

Experiment with transformations in the plane.

G-CO.2 Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).

G-CO.3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

G-CO.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

G-CO.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

Understand congruence in terms of rigid motions.

G-CO.6 Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

G-CO.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

Prove geometric theorems.

G-CO.11 Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.*

Common Core State Standards for Mathematical Practice

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Clarifying the Standards

Prior Learning

In sixth grade, students drew polygons in the coordinate plane given coordinates for the vertices. They used the coordinates to find the length of a side, joining points for the first coordinate point. They applied these techniques in the context of solving real-world and mathematical problems. In the eighth grade, students understood that a two-dimensional figure is congruent to another if the second figure can be obtained from the first by a sequence of rotations, reflections, or translations.

Current Learning

In this unit, students establish triangle congruence criteria based on analyses of rigid motions and formal constructions. They use triangle congruence as a familiar foundation for the development of a formal proof.

Future Learning

In algebra 2 and precalculus, students will expand their knowledge of transformations when building functions from existing functions. They may use their prior knowledge and skills in the professional fields of medical and health services, industrial production managers, insurance underwriters, purchasing managers, buyers, and purchasing agents.

Additional Findings

The work in this unit will be challenging, since students will apply reasoning to complete geometric constructions and explain why the constructions work. They will prove theorems using a variety of formats and solve problems about triangles, quadrilaterals, and other polygons.